

Transient acceleration in $f(T)$ gravity

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Abstract. Recently a $f(T)$ gravity based on the modification of the teleparallel gravity was proposed to explain the accelerated expansion of the universe without the need of dark energy. We use observational data from Type Ia Supernovae, Baryon Acoustic Oscillations, and Cosmic Microwave Background to constrain this $f(T)$ theory and reconstruct the effective equation of state and the deceleration parameter. We obtain the best-fit values of parameters and find an interesting result that the $f(T)$ theory considered here allows for the accelerated Hubble expansion to be a transient effect.

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1 Introduction

A series of independent cosmological observations including the type Ia supernovae (SNIa) [1], large scale structure [2], baryon acoustic oscillation (BAO) peaks [3] and cosmic microwave background (CMB) anisotropy [4] have probed the accelerating expansion of the universe. Subsequently, many gravitational theories and cosmological models were proposed to explain this cosmological phenomenon. Under the assumption of cosmological principle, these theories include the mysterious dark energy with negative pressure in general relativity and modified gravity models to the general relativity. For the former, the acceleration is realized by the drive of exotic dark energy, such as the cosmological constant, quintessence or phantom. The cosmological constant model (Λ CDM) is the simplest candidate for dark energy model, and agrees well with current cosmological observations. However, the Λ CDM model is immersed in the fine-tuning problem [5, 6] and coincidence problem [7]. Moreover, the nature of dark energy in form of other candidates still cannot be revealed. For the latter, the acceleration is realized by modification to the general relativity without exotic dark energy, such as the brane-world Dvali-Gabadadze-Porrati model [8], $f(R)$ gravity [9], Gauss-Bonnet gravity [10].

Similar as the exotic dark energy and other modified gravity models, it is found that the cosmic acceleration can also be obtained successfully from another gravitational scenario $f(T)$ theory [11]. It was proposed based on the Teleparallel Equivalent of General Relativity which is also known as Teleparallel Gravity. A scalar T is the Lagrangian of teleparallel gravity. The teleparallel gravity is not a new theory of gravity, but an alternative geometric formulation of general relativity. In teleparallel gravity, the Levi-Civita connection used in Einstein's general relativity is replaced by the Weitzenböck connection with torsion. However, the torsion vanishes in the dark energy and modified gravity models. Moreover, $f(T)$ theories have several interesting features. They not only can explain the late accelerating expansion, but always have second order differential equations, which is simpler than the $f(R)$ gravity. In addition, when certain conditions are satisfied, the behavior of $f(T)$ will be similar to quintessence [12]. Up to now, a number of $f(T)$ theories have been proposed [11, 13–17]. Under these cases, Yang [18] found that $f(T)$ theories are not dynamically equivalent to

teleparallel action plus a scalar field. Like other gravity theories and models, the $f(T)$ theories also have been investigated using the popular observational data. Investigations show that the $f(T)$ theories are compatible with observations (see e.g. [19, 20] and references therein). So, we note that the new type of $f(T)$ theory in [14] was proposed to explain the acceleration. It behaves like a cosmological constant, but is free from the coincidence problem. However, observational analysis in this model is still absent. Here, we would like to perform some further analysis using the observational data, such as the SNIa, BAO, and CMB.

This paper is organized as follows. In Sec 2, the general $f(T)$ gravity and the $f(T)$ model proposed in [14] are introduced. In Sec 3, we describe the method for constraining cosmological models and reconstruction scheme. And then the parameters of the specific $f(T)$ model are constrained by observational data. In addition, through the reconstruction scheme the effective equation of state and the deceleration parameter are reconstructed in Sec 4. Finally, we give the summary and conclusions in Sec 5.

2 The $f(T)$ theory

The $f(T)$ theory is a modification of teleparallel gravity, which uses the curvatureless Weitzenböck connection instead of torsionless Levi-Civita connection in Einstein's General Relativity. The curvatureless torsion tensor is

$$T^\lambda_{\mu\nu} \equiv e^\lambda_i (\partial_\mu e^i_\nu - \partial_\nu e^i_\mu), \quad (2.1)$$

where e^μ_i ($\mu = 0, 1, 2, 3$) are components of the four linearly independent vierbein field $\mathbf{e}_i(x^\mu)$ ($i = 0, 1, 2, 3$) in a coordinate basis. In particular, the vierbein is an orthonormal basis for the tangent space at each point x^μ of the manifold: $\mathbf{e}_i \cdot \mathbf{e}_j = \eta_{ij}$, where $\eta_{ij} = \text{diag}(1, -1, -1, -1)$. Notice that Latin indices refer to the tangent space, while Greek indices label coordinates on the manifold. The metric tensor is obtained from the dual vierbein as $g_{\mu\nu}(x) = \eta_{ij} e^i_\mu(x) e^j_\nu(x)$. The torsion scalar is the Lagrangian of teleparallel gravity [11]

$$T \equiv S_\rho{}^{\mu\nu} T^\rho_{\mu\nu}, \quad (2.2)$$

where

$$S_\rho{}^{\mu\nu} = \frac{1}{2} \left(K^{\mu\nu}_\rho + \delta^\mu_\rho T^{\theta\nu}_\theta - \delta^\nu_\rho T^{\theta\mu}_\theta \right), \quad (2.3)$$

and the contorsion tensor $K^{\mu\nu}_\rho$ is given by

$$K^{\mu\nu}_\rho = -\frac{1}{2} \left(T^{\mu\nu}_\rho - T^{\nu\mu}_\rho - T_\rho{}^{\mu\nu} \right). \quad (2.4)$$

In the $f(T)$ theory, we allow the Lagrangian density to be a function of T [11, 13, 21], thus the action reads

$$I = \frac{1}{16\pi G} \int d^4x e f(T), \quad (2.5)$$

where $e = \det(e^i_\mu) = \sqrt{-g}$. The corresponding field equation is

$$[e^{-1} \partial_\mu (e S_i{}^{\mu\nu}) - e_i{}^\lambda T^\rho_{\mu\lambda} S_\rho{}^{\nu\mu}] f_T + S_i{}^{\mu\nu} \partial_\mu T f_{TT} + \frac{1}{4} e_i{}^\nu f(T) = \frac{1}{2} k^2 e_i{}^\rho T_\rho{}^\nu, \quad (2.6)$$

where $k^2 = 8\pi G$, $f_T \equiv df/dT$, $f_{TT} \equiv d^2f/dT^2$, $S_i{}^{\mu\nu} \equiv e_i{}^\rho S_\rho{}^{\mu\nu}$, and $T_{\mu\nu}$ is the matter energy-momentum tensor. Obviously, Eq.(2.6) is a second-order equation. Thus, the $f(T)$ theories are simpler than the $f(R)$ theories with fourth-order equations.

Considering a flat homogeneous and isotropic FRW universe, we have

$$e_\mu^i = \text{diag}(1, a(t), a(t), a(t)) \quad , \quad e_i^\mu = \text{diag}\left(1, \frac{1}{a(t)}, \frac{1}{a(t)}, \frac{1}{a(t)}\right), \quad (2.7)$$

where $a(t)$ is the cosmological scale factor. By substituting Eqs.(2.7), (2.1), (2.3) and (2.4), into Eq. (2.2), we obtain the torsion scalar [11]

$$T \equiv S^{\rho\mu\nu}T_{\rho\mu\nu} = -6 H^2, \quad (2.8)$$

where H is the Hubble parameter $H = \dot{a}/a$. The dot represents the first derivative with respect to the cosmic time. Substituting Eq. (2.7) into (2.6), one can obtain the corresponding Friedmann equations

$$12H^2 f_T + f = 2k^2 \rho, \quad (2.9)$$

$$48H^2 \dot{H} f_{TT} - (12H^2 + 4\dot{H})f_T - f = 2k^2 p, \quad (2.10)$$

with ρ and p are the total energy density and pressure, respectively. The detailed calculation can be found in Ref. [11]. The conservation equation reads

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (2.11)$$

We should note that the total components considered here are matter and radiation, without dark energy. After brief simplification to the Friedmann Eqs.(2.9) and (2.10), we can rewrite them as

$$\frac{3}{k^2}H^2 = \rho + \rho_{\text{eff}}, \quad (2.12)$$

$$\frac{1}{k^2}(2\dot{H} + 3H^2) = -(p + p_{\text{eff}}), \quad (2.13)$$

where the effective energy density ρ_{eff} and pressure p_{eff} contributed from torsion are respectively given by [14]

$$\rho_{\text{eff}} = \frac{1}{2k^2}(-12H^2 f_T - f + 6H^2), \quad (2.14)$$

$$p_{\text{eff}} = -\frac{1}{2k^2}[48\dot{H}H^2 f_{TT} - 4\dot{H}f_T + 4\dot{H}] - \rho_{\text{eff}}. \quad (2.15)$$

We term it “effective” because it is just a geometric effect instead of a specific cosmic component. Therefore, what we are interested in is the acceleration driven by the torsion, not the exotic dark energy. Using Eqs.(2.14) and (2.15), we can define the total and effective equation of state as [14]

$$w_{\text{tot}} \equiv \frac{p + p_{\text{eff}}}{\rho + \rho_{\text{eff}}} = -1 + \frac{2(1+z)}{3H} \frac{dH}{dz}, \quad (2.16)$$

$$w_{\text{eff}} \equiv \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = -1 - \frac{48\dot{H}H^2 f_{TT} - 4\dot{H}f_T + 4\dot{H}}{-12H^2 f_T - f + 6H^2}. \quad (2.17)$$

The deceleration parameter, as usual, is defined as

$$q(z) \equiv -\frac{\ddot{a}}{aH^2} = -1 + \frac{(1+z)}{H} \frac{dH}{dz}. \quad (2.18)$$

After reviewing the general formation of $f(T)$ gravity, we now focus on a type of $f(T)$ gravity proposed in Ref. [14]

$$f(T) = T - \alpha T_0 \left[\left(1 + \frac{T^2}{T_0^2} \right)^{-n} - 1 \right], \quad (2.19)$$

which is analogy with a type of $f(R)$ theory proposed in Ref. [22], where α and n are positive constants. $T_0 = -6H_0^2$ and H_0 is the current value of the Hubble parameter. This type of $f(T)$ gravity has attracted much attention and has been discussed in detail in Ref. [23]. Here we will consider observational constraints on this type of $f(T)$ gravity. For $f(T)$ taking the form of Eq. (2.19), Eq.(2.9) can be rewritten as

$$E^2 + \frac{4n\alpha E^4}{(1 + E^4)^{n+1}} + \frac{\alpha}{(1 + E^4)^n} - B = \alpha, \quad (2.20)$$

where $E^2 \equiv H^2/H_0^2$ and $B = \Omega_{m0}(1+z)^3$, where Ω_{m0} is the matter density parameter today. Here we only focus on the evolution of the universe at low redshift, so we neglect the contribution of radiation. For $E(z=0) = 1$, we have $\alpha = (1 - \Omega_{m0})/(1 - 2^{-n+1}n - 2^{-n})$. This type of $f(T)$ model has some interesting characteristics. Firstly, the cosmological constant is zero in the flat space-time because $f(T=0) = 0$, while the geometrical one attributes as the dark energy. Secondly, it can behave like the cosmological constant. Such characteristics indicate that this type of $f(T)$ model is possible to be admitted by observational data, while impossible to be distinguished from the Λ CDM. Moreover, though the behavior of this type of $f(T)$ theory is similar to Λ CDM because of its dynamic behavior, it can relieve the coincidence problem suffered by Λ CDM.

3 Observational data and fitting method

In this section, we would like to introduce the observational data and constraint method. The corresponding observational data here are distance moduli of SNIa, CMB shift parameter and BAO distance parameter.

3.1 Type Ia supernovae

As early as 1998, cosmic accelerating expansion was first observed by "standard candle" SNIa which has the same intrinsic luminosity. Therefore, the observable is usually presented in the distance modulus, the difference between the apparent magnitude m and the absolute magnitude M . The latest version is Union2.1 compilation [24] which includes 580 samples. They are discovered by the Hubble Space Telescope Cluster Supernova Survey over the redshift interval $0.01 < z < 1.42$. The theoretical distance modulus is given by

$$\mu_{th}(z) = m - M = 5 \log_{10} D_L(z) + \mu_0, \quad (3.1)$$

where $\mu_0 = 42.38 - 5 \log_{10} h$, and h is the Hubble constant H_0 in the units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The corresponding luminosity distance function $D_L(z)$ is

$$D_L(z) = (1+z) \int_0^z \frac{dz'}{E(z'; \mathbf{p})}, \quad (3.2)$$

where $E(z'; \mathbf{p})$ is the dimensionless Hubble parameter given by Eq.(2.20), and \mathbf{p} stands for the parameter vector of the evaluated model embedded in expansion rate parameter $E(z)$.

We note that parameters in the expansion rate $E(z)$ include the annoying parameter h . In order to be immune from the Hubble constant, we should marginalize over the nuisance parameter μ_0 by integrating the probabilities on μ_0 [25–27]. Finally, we can estimate the remaining parameters without h by minimizing

$$\tilde{\chi}_{\text{SN}}^2(z, \mathbf{p}) = A - \frac{B^2}{C}, \quad (3.3)$$

where

$$\begin{aligned} A(\mathbf{p}) &= \sum_i \frac{[\mu_{\text{obs}}(z) - \mu_{\text{th}}(z; \mu_0 = 0, \mathbf{p})]^2}{\sigma_i^2(z)}, \\ B(\mathbf{p}) &= \sum_i \frac{\mu_{\text{obs}}(z) - \mu_{\text{th}}(z; \mu_0 = 0, \mathbf{p})}{\sigma_i^2(z)}, \\ C &= \sum_i \frac{1}{\sigma_i^2(z)}, \end{aligned}$$

and μ_{obs} is the observational distance modulus. This approach has been used in the reconstruction of dark energy [28], parameter constraint [29], reconstruction of the energy condition history [30] etc.

3.2 Cosmic microwave background

The CMB experiment measures the temperature and polarization anisotropy of the cosmic radiation in early epoch. It generally plays a major role in establishing and sharpening the cosmological models. In the CMB measurement, the shift parameter R is a convenient way to quickly evaluate the likelihood of the cosmological models. It contains the main information of the CMB observation [31, 32]. It is expressed as

$$R = \sqrt{\Omega_{m0}} \int_0^{z_s} \frac{dz'}{E(z'; \mathbf{p})}, \quad (3.4)$$

where $z_s = 1090.97$ is the redshift of decoupling. According to the measurement of WMAP-9 [33], we estimate the parameters by minimizing the corresponding χ^2 statistics

$$\chi_R^2 = \left(\frac{R - 1.728}{0.016} \right)^2. \quad (3.5)$$

3.3 Baryon acoustic oscillation

The measurement of BAO in the large-scale galaxies has rapidly become one of the most important observational pillars in cosmological constraints. This measurement is usually called the standard ruler in cosmology [34]. The distance parameter A obtained from the BAO peak in the distribution of SDSS luminous red galaxies [3] is a significant parameter and is defined as

$$A_{\text{th}} = \Omega_{m0}^{1/2} E(z_1)^{-1/3} \left[\frac{1}{z_1} \int_0^{z_1} \frac{dz'}{E(z'; \mathbf{p})} \right]^{2/3}. \quad (3.6)$$

We use the three combined data points in Ref. [35] that cover $0.1 < z < 2.4$ to determine the parameters in evaluated models. The expression of χ^2 statistics is

$$\chi_A^2 = \sum_i \left(\frac{A_{\text{th}} - A_{\text{obs}}}{\sigma_A^2} \right)^2, \quad (3.7)$$

where A_{obs} is the observational distance parameter and σ_A is its corresponding error.

Since the SNIa, CMB, and BAO data points are effectively independent measurements, we can simply minimize their total χ^2 values

$$\chi^2(z, \mathbf{p}) = \tilde{\chi}_{\text{SN}}^2 + \chi_{\text{R}}^2 + \chi_{\text{A}}^2,$$

to determine the parameters in the evaluated $f(T)$ model.

3.4 Reconstructing method

Using the above introduced χ^2 statistics, we can obtain the best-fit values and their errors of basic parameters \mathbf{p} . Further, we can reconstruct the other variable F relative with the known basic parameters \mathbf{p} by error propagation following the method in Ref. [36]. For example, the estimation from the observational data on the i th parameter p_i is $p_i = p_{0i}^{+\sigma_{iu}}_{-\sigma_{il}}$, where p_{0i} is the best-fit value, σ_{iu} and σ_{il} are the upper limit and lower limit, respectively. Errors of the reconstructed function F are estimated by

$$\begin{aligned} \delta F_u &= \sqrt{\sum_i \left[\max\left(\frac{\partial F}{\partial p_i} \sigma_{iu}, -\frac{\partial F}{\partial p_i} \sigma_{il}\right) \right]^2}, \\ \delta F_l &= \sqrt{\sum_i \left[\min\left(\frac{\partial F}{\partial p_i} \sigma_{iu}, -\frac{\partial F}{\partial p_i} \sigma_{il}\right) \right]^2}, \end{aligned} \quad (3.8)$$

where δF_u and δF_l are its upper and lower bound, respectively. In this paper, we will use this method to reconstruct the effective equation of state w_{eff} and deceleration parameter q .

4 Constraint result

Using the observational data sets, we perform the χ^2 statistics and display the contour constraint in Figure 1. We find that the combined data give mild constraints on them, i.e., $\Omega_{m0} = 0.22^{+0.0089}_{-0.0094}(1\sigma)$ and $n = 7.64^{+1.1750}_{-0.6700}(1\sigma)$ with $\chi^2_{\text{min}} = 579.4786$. If we consider the degrees of freedom (dof), the result is also good, i.e., $\chi^2_{\text{min}}/\text{dof} = 0.9923$, which indicates that this $f(T)$ model is consistent well with the observations. However, we note that the parameter n is worse at 95.4% confidence level. Namely, n is larger than 6. If the parameter n tends to infinite, we find from Eq. (2.19) that this $f(T)$ model finally recovers to the standard Λ CDM model.

In terms of Eq. (3.8), we reconstruct the effective equation of state in Figure 2. We find that $w_{\text{eff}}(z)$ is a decreasing function of redshift, and steadily approaches to -1 for high redshift $z \gtrsim 1$. That is, the geometric effect behaves like the cosmological constant at early epoch. However, it generally increases with the decrease of redshift. The present value of the effective equation of state finally reaches $w_{\text{eff}0} = -0.8760$. Moreover, the $w_{\text{eff}}(z)$ crosses through -1 for $z < 1$ within 1σ confidence level. In Figure 3, we also reconstruct the deceleration parameter $q(z)$. We find that the transition from decelerating to accelerating expansion occurs at $z = 0.95 \pm 0.05$, which is earlier than some phenomenological deceleration parameters [37, 38]. With the decrease of deceleration parameter, its value today is $q_0 = -0.3750$. In the near future $z = -0.04$, the $q(z)$ crosses the zero. That is to say, the accelerating expansion of the universe may be slowing down again and tends to decelerating expansion in future. It is possible, however, to have an eternal accelerated phase at 68.3% confidence level as shown

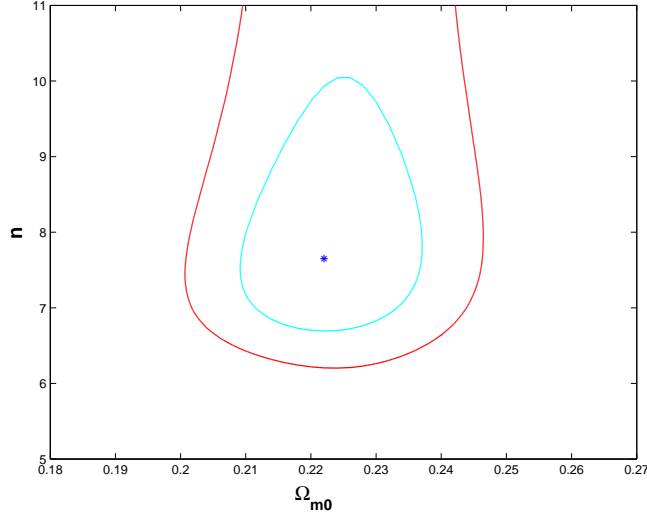


Figure 1. Constraints on $f(T)$ theory with 68.3% and 95.4% confidence regions in the Ω_{m0} - n plane fitting from combinational observation of SNIa, BAO, and CMB data. The blue asterisk is the best-fit point.

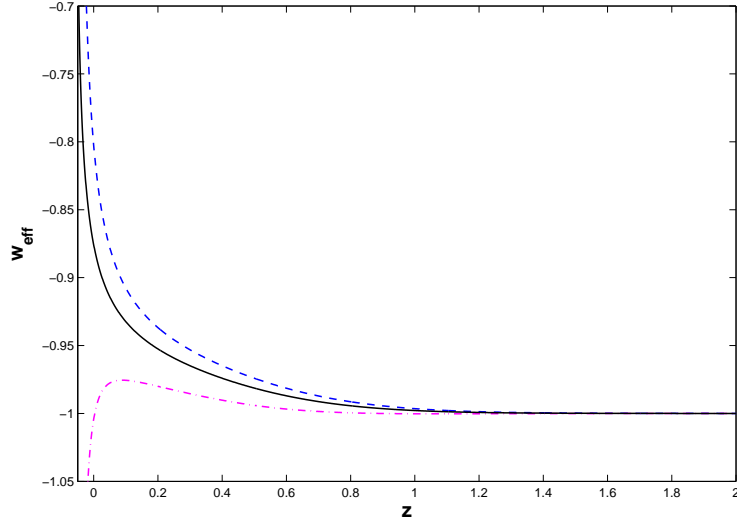


Figure 2. Reconstruction of the effective equation of state at 68.3% confidence level for the $f(T)$ gravity considered here.

in Figure 3. The feature of transient acceleration makes the $f(T)$ gravity considered here compatible with the S-matrix description of string theory [39, 40]. Recently it was also argued that the SNIa data could favor a transient acceleration [41]. It was also pointed out in [42–44] that a transient phase of accelerated expansion is not excluded by current observations.

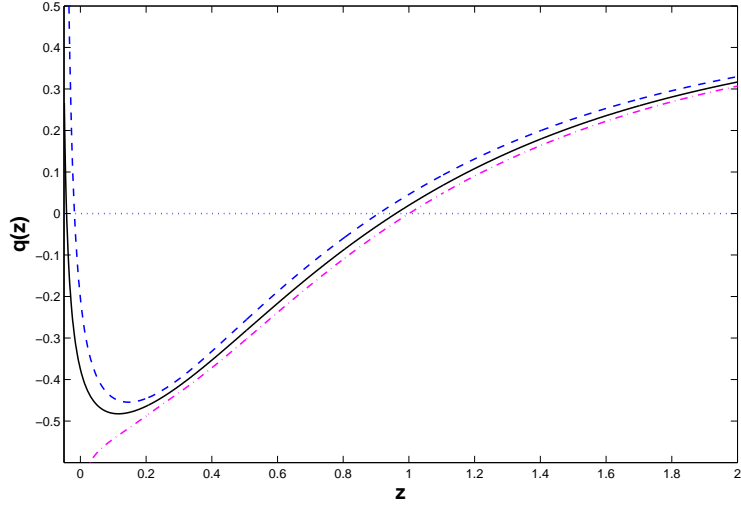


Figure 3. Reconstruction of the deceleration parameter at 68.3% confidence level for the $f(T)$ gravity considered here.

5 Summary and conclusions

The $f(T)$ gravity which is based on modification of the teleparallel gravity was proposed to explain the accelerating expansion of the universe without the need of dark energy. We gave a brief overview of a specific $f(T)$ gravity proposed in [14]. We also introduced the method used to constrain cosmological models with observational data including SNIa, BAO, and CMB. After constraining the $f(T)$ gravity proposed in [14], we find that the best-fit values of the parameters at the 68.3% confidence level are: $\Omega_{m0} = 0.22^{+0.0089}_{-0.0094}$ and $n = 7.64^{+1.175}_{-0.67}$ with $\chi^2_{\min} = 579.4786$ ($\chi^2_{\min}/\text{dof} = 0.9923$). The parameters Ω_{m0} and n can be constrained well at 68.3% confidence level by these observational data.

We also reconstructed the effective equation of state and the deceleration parameter from observational data. We found that the transition from deceleration to acceleration occurs at $z = 0.95 \pm 0.05$. The present value of deceleration parameter was found to be $q_0 = -0.3750$, meaning that the cosmic expansion has passed a maximum value (about at $z \sim 0.1$) and is now slowing down again. This is a theoretically interesting result because eternally accelerating universe (like Λ CDM) is endowed with a cosmological event horizon which prevents the construction of a conventional S-matrix describing particle interactions. Such a difficulty has been pointed out as a severe theoretical problem for any eternally accelerated universe [40, 45, 46]. Some researches also indicated that a transient phase of accelerated expansion is not excluded by current observations [42–44]. We note, however, it is possible to have an eternal accelerated phase and an effective equation of state crossing through -1 at 68.3% confidence level, according to the reconstruction of the effective equation of state and the deceleration parameter.

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